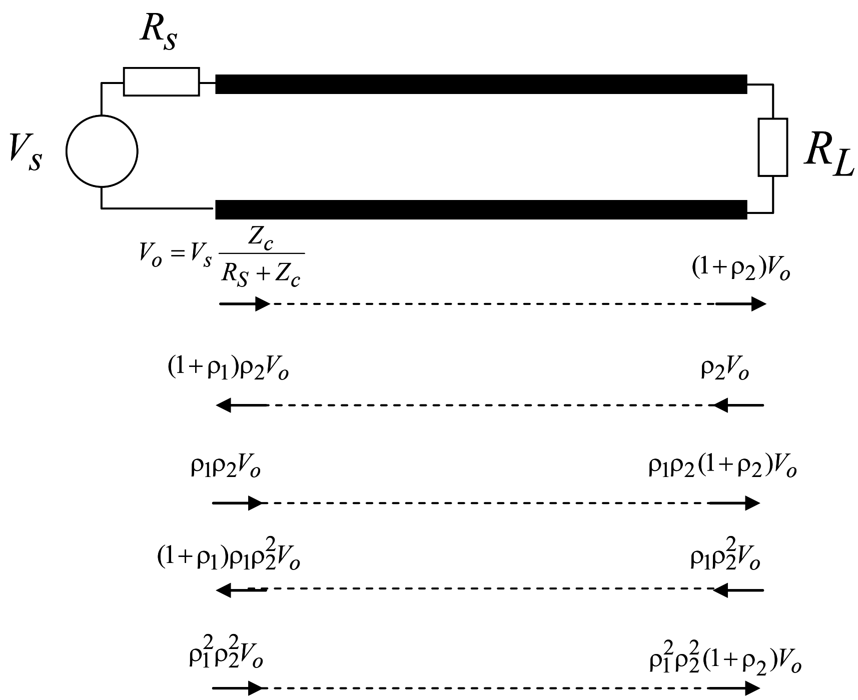
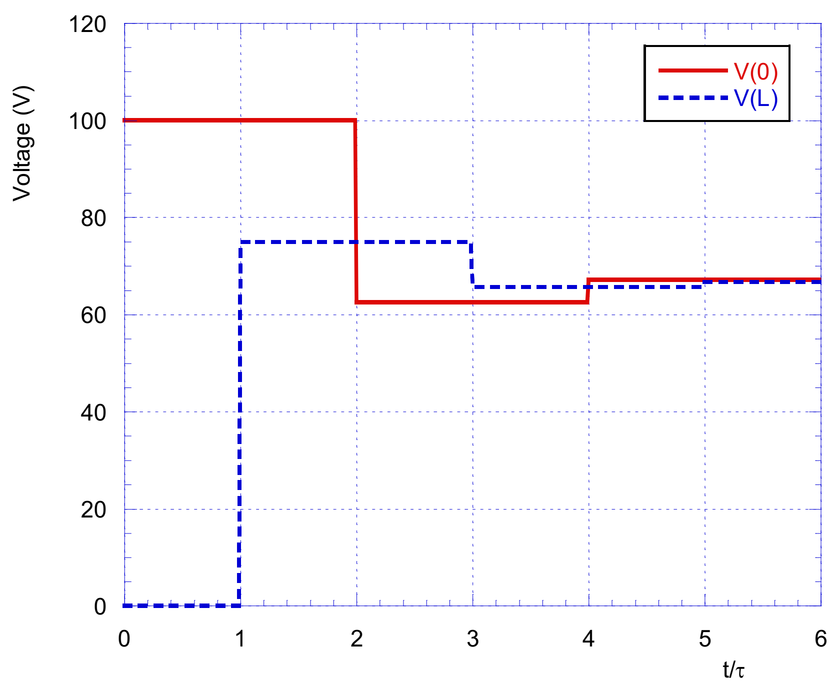
Electromagnetic Compatibility
Problem Set II – Solutions

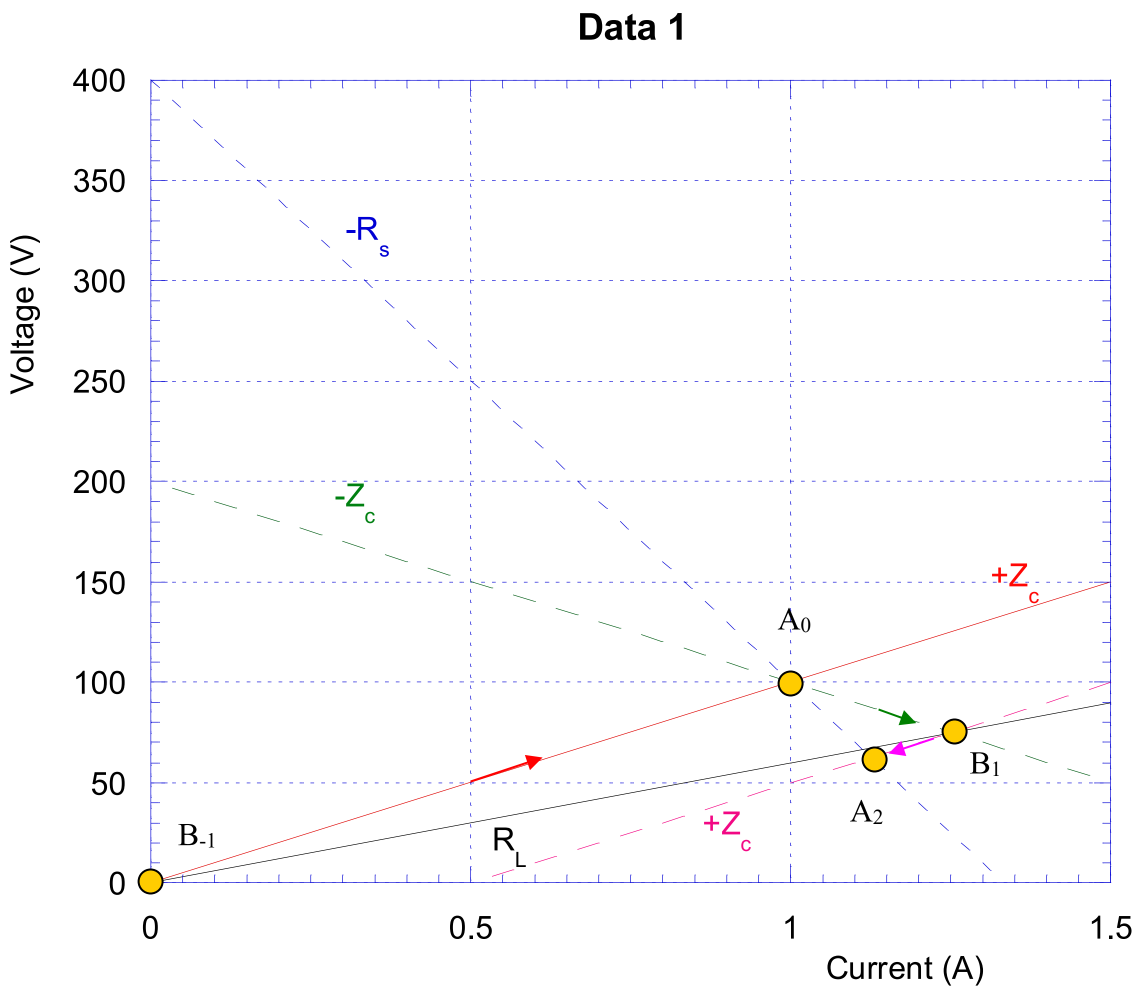
1.



$$\rho_1 = \frac{R_s - Z_c}{R_s + Z_c} = 0.5 \qquad \rho_2 = \frac{R_L - Z_c}{R_L + Z_c} = -0.25$$



Bergeron's diagram:



2. $Z_c = 50\ \Omega$, $R_s = 50\ \Omega$.

$$v = \frac{c}{\sqrt{\varepsilon_r}} = 2 \cdot 10^8 \text{ m/s},\ 2\tau = 6\ \mu s \Rightarrow \tau = 3\ \mu s$$

$$L = v\tau = 600 \text{ m}$$

$$V_s = 200 \text{ V}$$

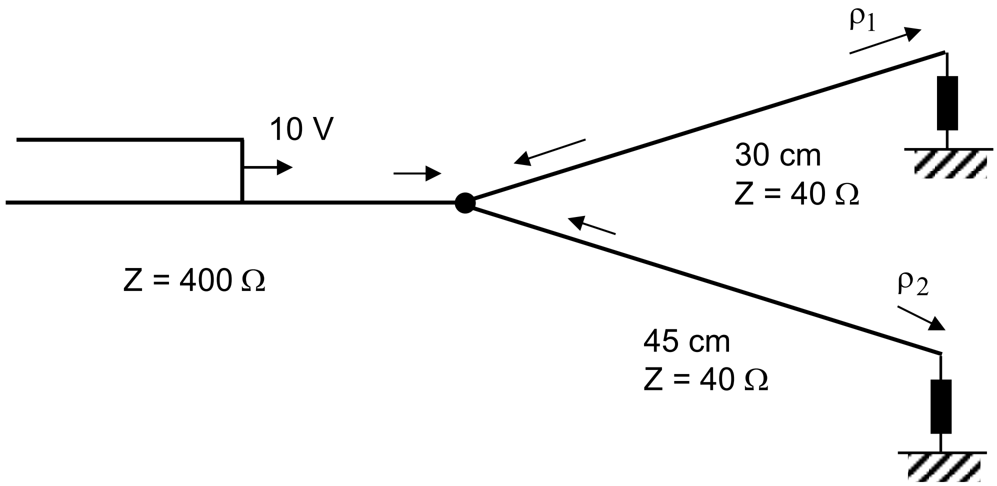
$$\rho_2 = 0.2 = \frac{R_L - 50}{R_L + 50} \quad \Rightarrow \quad R_L = 75\Omega$$

3. $Z_c = \dfrac{12}{0.150} = 80\ \Omega$

$$\rho_1 = -1,\ \rho_2 = \frac{R_L - Z_c}{R_L + Z_c}$$

$$-160\ mA = 150\ mA(-\rho_2)(1-\rho_1) \quad \Rightarrow \quad \rho_2 = 0.53 \quad \Rightarrow \quad R_L = 260.4\ \Omega$$

4.



$$\rho_o = \frac{20-400}{20+400} = -0.9$$

$$\rho_1 = -1$$

$$\rho_2 = \frac{20-40}{20+40} = -0.33$$

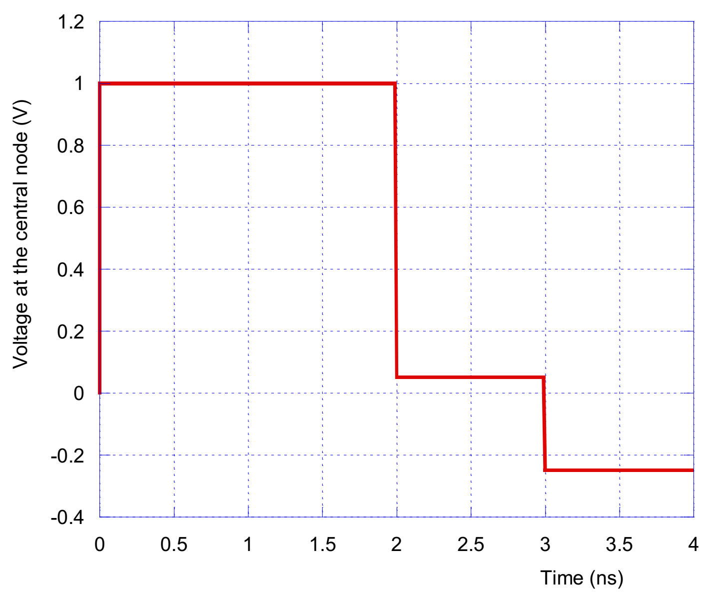
$$\rho_3 = \rho_4 = \frac{36.36-40}{36.36+40} = -0.05$$

$$\tau_1 = 1\,\text{ns}, \quad \tau_2 = 1.5\,\text{ns}$$

$$V_o = (1+\rho_o)10 = 1\text{ V}$$

$$\Delta V_1 = \rho_1(1+\rho_3)V_o = -0.95\text{ V}$$

$$\Delta V_2 = \rho_2(1+\rho_3)V_o = -0.3\text{ V}$$



Note that ρ_o, ρ_3, and ρ_4 are the 'initial' reflection coefficients seen by the front of the initial wave.

5. a

$V = V^{+} + V^{-} \quad V = V^{++}$

$I^{+} + I^{-} = I_c + I^{++} \quad \Leftrightarrow \quad \frac{V^{+}}{Z_c} - \frac{V^{-}}{Z_c} = I_c + \frac{V^{++}}{Z_c} \quad \Leftrightarrow \quad V^{+} - V^{-} = Z_c I_c + V^{++}$ (1)

$V^{+} + V^{-} = V^{++}$ (2)

$I_c = C\frac{dV^{++}}{dt}$ (3)

(2)-(1): $V^{-} = -\frac{Z_c I_c}{2}$

(2)+(1), taking into account (3): $\frac{dV^{++}}{dt} + \frac{2}{Z_c C}V^{++} = \frac{2V^{+}}{Z_c C}$

General solution:

$V^{++}(t) = V^{+}(1 - e^{-t/\tau}), \quad \tau = \frac{Z_c C}{2}$

$0.9 = 1 - e^{-t/\tau} \quad \Rightarrow \quad t = 1.15 Z_c C = 57.5\, ps$

5.b

$I^{++} = I^{+} + I^{-} \quad \Leftrightarrow \quad \frac{V^{++}}{Z_c} = \frac{V^{+}}{Z_c} - \frac{V^{-}}{Z_c} \quad \Leftrightarrow \quad V^{++} = V^{+} - V^{-}$ (1)

$V^{+} + V^{-} = L\frac{dI^{++}}{dt} + V^{++} = \frac{L}{Z_c}\frac{dV^{++}}{dt} + V^{++}$ (2)

(1)+(2): $2V^{+} = \frac{L}{Z_c}\frac{dV^{++}}{dt} + V^{++} \quad \Leftrightarrow \quad \frac{dV^{++}}{dt} + \frac{2Z_c}{L}V^{++} = \frac{2Z_c}{L}V^{+}$

General Solution: $V^{++}(t) = V^{+}(1 - e^{-t/\tau}), \quad \tau = \frac{L}{2Z_c}$

$0.9 = 1 - e^{-t/\tau} \quad \Rightarrow \quad t = 1.15\frac{L}{Z_c} = 57.5\, ps$